Spectral- and Energy-Efficient Transmission Over Frequency-Orthogonal Channels

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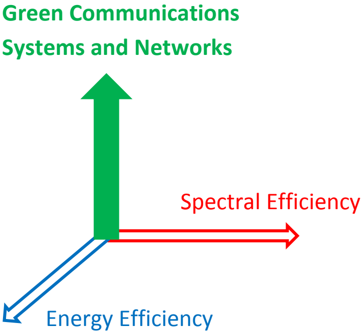
Introduction



- Spectral efficiency allows the network to maximize utilization of assigned frequencies to provide better services for more users.

  It is measured as the communication data rate per unit bandwidth used.

- Energy efficiency allows the network to minimize energy consumption for transferring a certain amount of information.

  It is measured as the communication data rate per unit of power.

Introduction

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4. The amount of total bandwidth and transmit power needs to be managed to maximize the spectral efficiency and the energy efficiency.

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3. The maximum communication data rate is characterized by the sum capacity of channels from the transmitter to the multiple receivers.
4. The amount of total bandwidth and transmit power needs to be managed to maximize the spectral efficiency and the energy efficiency.
5. There is a minimum rate requirement for each user.

Introduction

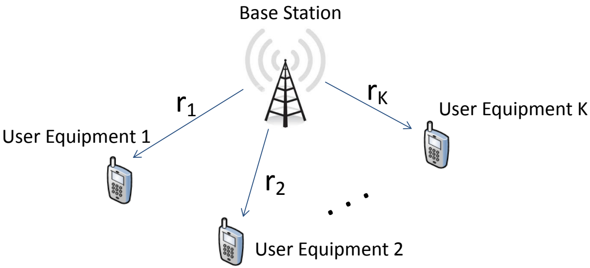
- Background:
  - Xiong et al. (2011) established a spectral efficiency–energy efficiency tradeoff framework in downlink OFDMA networks.
  - Deng et al. (2013) formulated the spectral efficiency–energy efficiency optimization problem as a multi-objective optimization problem and then converted it into a single-objective optimization problem.
  - Tang et al. (2014) proposed resource efficiency as a combined metric of spectral efficiency and energy efficiency in an OFDMA network with different transmission-bandwidth requirements.
  - Tsilimantos et al. (2016) studied the spectral efficiency-energy efficiency tradeoff in the cellular network downlink over orthogonal channels.

Introduction

- What is missing in the current research work is either a complete closed-form solution or a simple algorithm to find bandwidth and transmit power for maximum efficiency.

System Model

Frequency-orthogonal parallel broadcast:



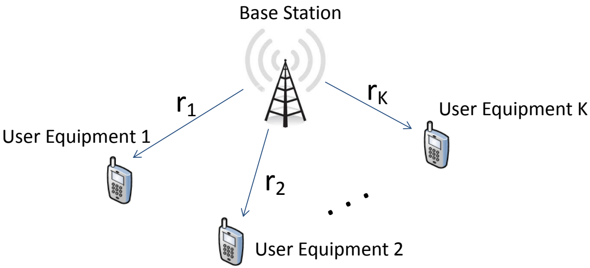
- Maximum achievable data rate ($\forall k \in \mathcal{K}$):

$$r_k = w_k \log_2\left(1 + \frac{p_k |h_k|^2}{w_k N_0 + I_k}\right) = w_k \log_2\left(1 + \frac{p_k g_k}{w_k}\right)$$

where g_k is the gain-to-interference-plus-noise-density ratio. g_k is the "channel quality" that is known.

System Model

Frequency-orthogonal parallel broadcast:



- The kth active UE has a minimum rate requirement R_k, such that

$$r_k \geq R_k, \ \forall k \in \mathcal{K}.$$

System Model

Spectral Efficiency

$$\Gamma_{\mathrm{SE}} \triangleq \frac{R}{W}$$

Energy Efficiency

$$\Gamma_{\mathrm{EE}} \triangleq \frac{R}{P}$$

- $R = \sum_{k\in\mathcal{K}} r_k$
- $W = \sum_{k\in\mathcal{K}} w_k \leq W_M$
  W is the total assigned bandwidth in the cell.
  W_M is the maximum allowed bandwidth for BS transmission.
- $P = \sum_{k\in\mathcal{K}} p_k \leq P_M$
  P is the total transmit power.
  P_M is the maximum transmit power that the BS can deliver.

Problem Formulation

Problem of spectral- and energy-efficient transmission

$$\mathcal{P}_1: \begin{array}{ll} \underset{W,P,\{w_k\},\{p_k\}}{\text{maximize}} & \Gamma_{\text{SE}}, \Gamma_{\text{EE}} \\ \text{subject to} & w_k \log_2\left(1+\frac{p_k g_k}{w_k}\right) \geq R_k, \ \ \forall k \in \mathcal{K} \\ & \sum_{k\in\mathcal{K}} w_k = W \leq W_M \\ & \sum_{k\in\mathcal{K}} p_k = P \leq P_M. \end{array}$$

A multi-objective optimization problem.

1. Optimization of bandwidth assignment with a fixed power allocation.
2. Optimization of transmit power allocation with a fixed bandwidth assignment.
3. Joint bandwidth assignment and transmit power allocation.

1a. Optimal Bandwidth Assignment

With Fixed Transmit Power Allocation

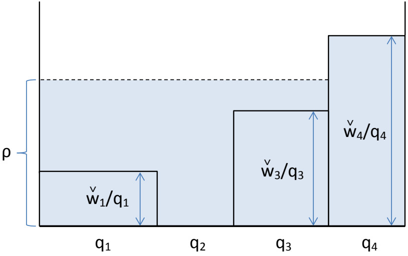
Problem of optimal bandwidth assignment

$$\mathcal{P}_2: \begin{array}{ll} \underset{\{w_k\}}{\text{maximize}} & \Gamma_{\text{SE}}(W) = \frac{1}{W}\sum_{k\in\mathcal{K}} w_k \log_2\left(1+\frac{p_k g_k}{w_k}\right) \\ \text{subject to} & w_k \log_2\left(1+\frac{p_k g_k}{w_k}\right) \geq R_k,\ \forall k\in\mathcal{K} \\ & \sum_{k\in\mathcal{K}} w_k = W \leq W_M. \end{array}$$

Optimal Solution:

$$\hat{w}_k = \check{w}_k + q_k\left(\rho - \frac{\check{w}_k}{q_k}\right)^+,\ \forall k\in\mathcal{K}$$

- $q_k = p_k g_k$
- $\check{w}_k$ minimum required bandwidth



1a. Optimal Bandwidth Assignment

- Define Set $\mathcal{I}$ — The set of UEs that are assigned with their minimum required bandwidths, i.e., $\hat{w}_i = \check{w}_i, \forall i \in \mathcal{I} \subseteq \mathcal{K}$.

The maximum spectral efficiency is

$$\hat{\Gamma}_{\mathrm{SE}}(W) = \begin{cases} R_0/W_0 & , \quad W = W_0 \\ \frac{1}{W}\left[\sum_{i\in\mathcal{I}} R_i + (W - \sum_{i\in\mathcal{I}} \check{w}_i)\log_2\left(1+\frac{1}{\rho}\right)\right] & , \quad W > W_0 \end{cases}$$

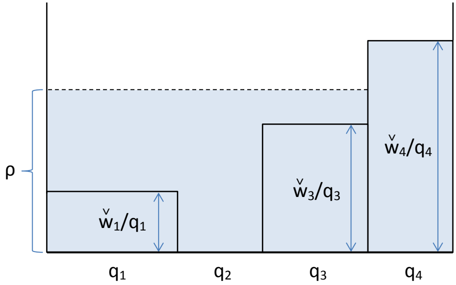
where

$$\begin{aligned} W_0 &= \sum_{k\in\mathcal{K}} \check{w}_k \\ R_0 &= \sum_{k\in\mathcal{K}} R_k \\ \rho &= \frac{W - \sum_{i\in\mathcal{I}} \check{w}_i}{\sum_{j\in\mathcal{K}\setminus\mathcal{I}} q_j} \end{aligned}$$

1a. Optimal Bandwidth Assignment

- Given a total bandwidth W, the optimal bandwidth assignment can be calculated as

$$\begin{aligned} \hat{w}_i &= \check{w}_i, \ \forall i \in \mathcal{I} \subseteq \mathcal{K} \\ \hat{w}_j &= \frac{q_j}{\sum_{j \in \mathcal{K} \setminus \mathcal{I}} q_j} \left(W - \sum_{i \in \mathcal{I}} \check{w}_i \right), \ \forall j \in \mathcal{K} \setminus \mathcal{I}. \end{aligned}$$



1a. Optimal Bandwidth Assignment

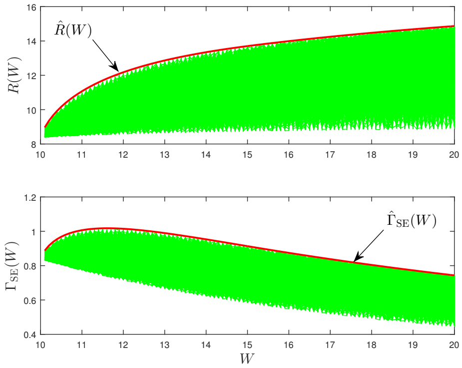


Figure: Sum rate and spectral efficiency with various bandwidth assignments and a fixed transmit power allocation.

- $\hat{\Gamma}_{\mathrm{SE}}(W)$ is continuously differentiable, and it is either strictly decreasing or strictly quasiconcave in $W \geq W_0$.
- The maximum achievable sum rate $\hat{R}(W) = W\hat{\Gamma}_{\mathrm{SE}}(W)$ is continuously differentiable, strictly increasing and concave in $W \geq W_0$.

1b. Find Total Bandwidth for Best Spectral Efficiency

- As $\hat{\Gamma}_{\mathrm{SE}}(W)$ is continuously differentiable, its first derivative is given by

$$\frac{d\hat{\Gamma}_{\mathrm{SE}}(W)}{dW} = \frac{d}{dW}\left(\frac{\hat{R}(W)}{W}\right) = \frac{d\hat{R}(W)}{dW}\frac{1}{W} - \frac{\hat{R}(W)}{W^2}$$

$$= \begin{cases} \frac{1}{W_0}\frac{1}{\ln 2}\Phi(\rho_0) - \frac{1}{W_0^2}R_0 & , \quad W = W_0 \\ \frac{1}{W^2}\left[\sum_i \check{w}_i \log_2\left(1 + \frac{\sum_j q_j}{W - \sum_i \check{w}_i}\right) \right. \\ \left. \quad - \sum_i R_i - \frac{W\sum_j q_j}{(\sum_j q_j + W - \sum_i \check{w}_i)\ln 2}\right] & , \quad W > W_0 \end{cases}$$

where

$$\Phi(x) = \ln(1 + 1/x) - 1/(1 + x)$$
$$\rho_0 = \min_{k \in \mathcal{K}}(\check{w}_k / q_k).$$

1b. Find Total Bandwidth for Best Spectral Efficiency

"Water-level" Analysis

- Define $\rho_k = \check{w}_k / q_k$ ($\forall k \in \mathcal{K}$) as the "water-level marks".
- Let $\{\rho_k\}_{k=1}^{K}$ be sorted in ascending order and denoted as $\rho'_1 \leq \rho'_2 \leq \cdots \leq \rho'_K$, where ρ'_k corresponds to $\check{w}'_k$, q'_k and R'_k, i.e., $\rho'_k = \check{w}'_k / q'_k$.
- The total bandwidth starts at the minimum level of $W_0 = \sum_{i=1}^{K} \check{w}'_i$.
- According to the bandwidth water-filling solution, the critical levels of W are the ones at which the water level reaches the water-level marks. In an ascending order, the critical levels are given by

$$W_{J-1} = \sum_{i=J+1}^{K} \check{w}'_i + \rho'_J \sum_{j=1}^{J} q'_j, \quad J = 1, 2, \ldots, K.$$

- When $W_{J-1} \leq W < W_J$, the $K - J$ UEs that correspond to $\rho'_{J+1}, \rho'_{J+2}, \ldots, \rho'_K$ are in Set $\mathcal{I}$.

1b. Find Total Bandwidth for Best Spectral Efficiency

- At the critical bandwidth levels $W_{J-1}, J = 1, 2, \ldots, K$, the maximum spectral efficiency and its derivative are given by

$$\hat{\Gamma}_{\mathrm{SE}}(W_{J-1}) = \frac{1}{W_{J-1}} \left(\sum_{i=J+1}^{K} R_i' + \frac{\sum_{j=1}^{J} q_j'}{q_J'} R_J' \right)$$

$$\left. \frac{d\hat{\Gamma}_{\mathrm{SE}}(W)}{dW} \right|_{W=W_{J-1}} = \frac{1}{W_{J-1}^2} \left(\frac{\sum_{i=J+1}^{K} \check{w}_i'}{\check{w}_J'} R_J' - \sum_{i=J+1}^{K} R_i' - \frac{W_{J-1}}{(1+\rho_J') \ln 2} \right).$$

1b. Find Total Bandwidth for Best Spectral Efficiency

- When $d\hat{\Gamma}_{\text{SE}}(W)/dW \mid_{W=W_{J-1}} \geq 0$ and $d\hat{\Gamma}_{\text{SE}}(W)/dW \mid_{W=W_J} < 0$, the optimal total bandwidth is in interval $[W_{J-1}, W_J)$.
- With the bisection method, the optimal total bandwidth W^{opt} can be found as the root of $\Theta(W)$

$$\Theta(W) = \check{W}_{\mathcal{I}} \ln\left(1 + \frac{Q}{W - \check{W}_{\mathcal{I}}}\right) - R_{\mathcal{I}} \ln 2 - \frac{WQ}{Q + W - \check{W}_{\mathcal{I}}}$$

where

$$\begin{aligned} \check{W}_{\mathcal{I}} &= \sum_{i=J+1}^{K} \check{w}'_i = \sum_{i \in \mathcal{I}} \check{w}_i, \quad Q = \sum_{j=1}^{J} q'_j = \sum_{j \in \mathcal{K} \setminus \mathcal{I}} q_j, \\ R_{\mathcal{I}} &= \sum_{i=J+1}^{K} R'_i = \sum_{i \in \mathcal{I}} R_i. \end{aligned}$$

Spectral-Efficient Transmission Algorithm

Find optimal total bandwidth and its assignment that maximize $\Gamma_{\rm SE}$

0. If there is no minimum rate requirement of any UE, i.e., $\check{w}_k = 0, \forall k \in \mathcal{K}$, the problem is invalid because $\hat{\Gamma}_{\rm SE}(W)$ is strictly decreasing in W. Otherwise, there are some non-zero $\{\check{w}_k\}$. Calculate $\check{w}_k$ from $R_k, \forall k \in \mathcal{K}$;

1. Calculate $\rho_k = \check{w}_k / q_k$, where $q_k = p_k g_k$;

2. Sort $\{\rho_k\}$ in ascending order as $\{\rho'_k\}_{k=1}^{K}$;

3. Check the sign of $d\hat{\Gamma}_{\rm SE}(W)/dW \mid_{W=W_0}$: If $\rho'_1 = 0$, the derivative is positive. Otherwise, the sign is $\mathrm{Sgn}[W_0 \Phi(\rho'_1) - R_0 \ln 2]$. If $d\hat{\Gamma}_{\rm SE}(W)/dW \mid_{W=W_0} \leq 0$, $W^{opt} = W_0$, go to Step 7;

Spectral-Efficient Transmission Algorithm

Find optimal total bandwidth and its assignment that maximize Γ_{SE}

4. Calculate the critical levels of total bandwidth $W_{J-1}, J = 1, 2, \ldots, K$. The bandwidth levels can be neglected if they are beyond W_M;

5. Calculate $d\hat{\Gamma}_{\mathrm{SE}}(W)/dW \mid_{W=W_{J-1}}$, starting with $J = 2$ and stopping when the derivative is negative;

6. When $d\hat{\Gamma}_{\mathrm{SE}}(W)/dW \mid_{W=W_{J-1}} \geq 0$ and $d\hat{\Gamma}_{\mathrm{SE}}(W)/dW \mid_{W=W_J} < 0$, the optimal total bandwidth is in interval $[W_{J-1}, W_J)$. Establish Set $\mathcal{I}$. Find W^{opt} as the root of $\Theta(W)$ using the bisection method;

7. With W^{opt}, calculate the optimal bandwidth assignment and the maximum spectral efficiency.

Simulation Results

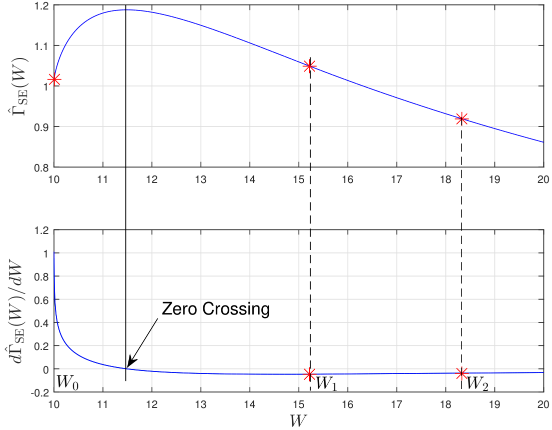


Figure: Search of the total bandwidth that maximizes the spectral efficiency. Fixed transmit power allocation.

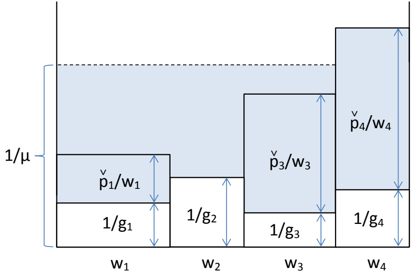
Problem of transmit power allocation

$$
\mathcal{P}_3: \quad \begin{aligned} \underset{\{p_k\}}{\text{maximize}} \quad & \Gamma_{\text{EE}}(P) = \frac{1}{P}\sum_{k\in\mathcal{K}} w_k \log_2\left(1+\frac{p_k g_k}{w_k}\right) \\ \text{subject to} \quad & p_k \geq \frac{w_k}{g_k}\left(2^{\frac{R_k}{w_k}}-1\right), \ \forall k\in\mathcal{K} \\ & \sum_{k\in\mathcal{K}} p_k = P \leq P_M. \end{aligned}
$$

Optimal Solution:

$$
\hat{p}_k = \check{p}_k + w_k\left(\frac{1}{\mu}-\alpha_k\right)^+, \ \forall k\in\mathcal{K}
$$

- $\check{p}_k = \frac{w_k}{g_k}\left(2^{\frac{R_k}{w_k}}-1\right)$
- $\alpha_k = \frac{1}{g_k}+\frac{\check{p}_k}{w_k}$ "water-level marks"



2a. Optimal Transmit Power Allocation

- Define Set $\mathcal{I}$ — The set that contains the UEs transmitting with their minimum required power, i.e., $\hat{p}_i = \check{p}_i, \forall i \in \mathcal{I} \subseteq \mathcal{K}$.

The maximum energy efficiency with a total transmit power P is given by

$$\boxed{\hat{\Gamma}_{\mathrm{EE}}(P) = \frac{1}{P}\left(\sum_{i\in\mathcal{I}} R_i + \sum_{j\in\mathcal{K}\setminus\mathcal{I}} w_j \log_2\left(\frac{g_j}{\mu}\right)\right)}$$

where

$$\begin{aligned} \frac{1}{\mu} &= \frac{P - P_0 + \sum_{j\in\mathcal{K}\setminus\mathcal{I}} w_j \alpha_j}{\sum_{j\in\mathcal{K}\setminus\mathcal{I}} w_j} \\ P_0 &= \sum_{k\in\mathcal{K}} \check{p}_k. \end{aligned}$$

2a. Optimal Transmit Power Allocation

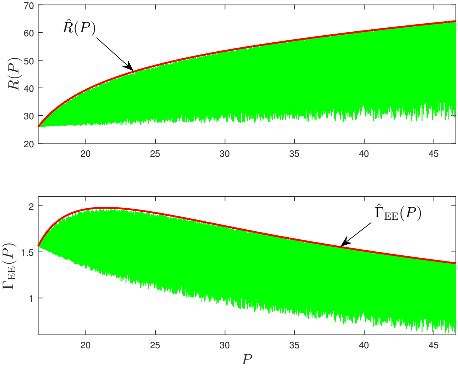


Figure: Sum rate and energy efficiency with various transmit power allocations and a fixed bandwidth assignment.

- $\hat{\Gamma}_{\mathrm{EE}}(P)$ is continuously differentiable, and
  it is either strictly decreasing or strictly quasiconcave in $P \geq P_0$.

2b. Find Total Transmit Power for Best Energy Efficiency

"Water-level" Analysis

- Let $\{\alpha_k\}_{k=1}^{K}$ be sorted in the ascending order and denoted as $\alpha'_1 \leq \alpha'_2 \leq \cdots \leq \alpha'_K$, where α'_k corresponds to g'_k, $\check{p}'_k$ and w'_k, i.e., $\alpha'_k = 1/g'_k + \check{p}'_k/w'_k$.

- As P increases from P_0, according to the transmit power water-filling solution, the critical levels of P are the ones at which the water level $1/\mu$ reaches $\alpha'_1, \alpha'_2, \ldots, \alpha'_K$ in its order.

- In an ascending order, the critical levels are given by

$$P_{J-1} = P_0 + \sum_{i=1}^{J-1} (\alpha'_J - \alpha'_i) w'_i, \;\; J = 1, 2, \ldots, K.$$

2b. Find Total Transmit Power for Best Energy Efficiency

- At the critical levels of total transmit power $P_{J-1}, J = 1, 2, \ldots, K$, the maximum energy efficiency is given by

$$\hat{\Gamma}_{\mathrm{EE}}(P_{J-1}) = \frac{R_0 + \sum_{i=1}^{J-1} w_i' \log_2(\alpha_J'/\alpha_i')}{P_0 + \sum_{i=1}^{J-1}(\alpha_J' - \alpha_i')w_i'}.$$

- The derivative of $\hat{\Gamma}_{\mathrm{EE}}(P)$ is given by

$$\frac{d\hat{\Gamma}_{\mathrm{EE}}(P)}{dP} = \frac{d}{dP}\left(\frac{\hat{R}(P)}{P}\right) = \frac{d\hat{R}(P)}{dP}\frac{1}{P} - \frac{\hat{R}(P)}{P^2}.$$

As $P > 0$, the sign of $d\hat{\Gamma}_{\mathrm{EE}}(P)/dP$ is determined by the sign of $\Lambda(P)$

$$\Lambda(P) = \frac{d\hat{R}(P)}{dP} - \hat{\Gamma}_{\mathrm{EE}}(P).$$

2b. Find Total Transmit Power for Best Energy Efficiency

- Let Λ_{J-1} denote $\Lambda(P_{J-1})$ with critical power level $P_{J-1}, J = 1, 2, \ldots, K$.

$$\Lambda_{J-1} = \frac{1}{P_{J-1}} \left(\frac{1}{\ln 2} \frac{1}{\alpha'_J} - \hat{\Gamma}_{\mathrm{EE}}(P_{J-1}) \right).$$

- If $\Lambda_0 \leq 0$, the optimal total transmit power to maximize energy efficiency is $P^{opt} = P_0$ with transmit power allocation $\hat{p}_k = \check{p}_k, \forall k \in \mathcal{K}$.

- In the situation when $\Lambda_{J-1} > 0$ and $\Lambda_J \leq 0$, J UEs with smallest α's are assigned excess transmit power beyond their minimum required power to achieve maximum energy efficiency.

  The optimal total transit power is in the interval $(P_{J-1}, P_J]$.

2b. Find Total Transmit Power for Best Energy Efficiency

- The maximum energy efficiency can be written as

$$\hat{\Gamma}_{\mathrm{EE}}(P) = \frac{1}{P}\left(A + \tilde{W}_J \log_2(P - B)\right), \quad P \in (P_{J-1}, P_J]$$

where

$$\begin{aligned} \tilde{W}_J &= \sum_{i=1}^{J} w_i' \\ A &= R_0 - \sum_{i=1}^{J} w_i' \log_2 \alpha_i' - \tilde{W}_J \log_2 \tilde{W}_J \\ B &= P_0 - \sum_{i=1}^{J} w_i' \alpha_i'. \end{aligned}$$

2b. Find Total Transmit Power for Best Energy Efficiency

- The derivative of $\hat{\Gamma}_{\mathrm{EE}}(P)$ in this interval is given by

$$\frac{d\hat{\Gamma}_{\mathrm{EE}}(P)}{dP} = \frac{\tilde{W}_J}{P(P-B)\ln 2} - \frac{A + \tilde{W}_J \log_2(P-B)}{P^2}.$$

- The sign of the derivative is determined by the sign of $\Theta(P)$

$$\Theta(P) = \frac{P\tilde{W}_J}{P-B} - A\ln 2 - \tilde{W}_J \ln(P-B).$$

- As $\Theta(P_{J-1}) > 0$ and $\Theta(P_J) \le 0$, the root of $\Theta(P), P \in (P_{J-1}, P_J]$ can be found using the bisection method.

Energy-Efficient Transmission Algorithm

Find optimal total transmit power and its allocation that maximize Γ_{EE}

0. Calculate minimum transmit power $\check{p}_k$ from minimum rate requirement $R_k, \forall k \in \mathcal{K}$;
1. Calculate $\alpha_k = \frac{1}{g_k} + \frac{\check{p}_k}{w_k}$;
2. Sort $\{\alpha_k\}$ in ascending order as $\{\alpha'_k\}_{k=1}^{K}$;
3. Calculate the critical levels of total transmit power $P_{J-1}, J = 1, 2, \ldots, K$. The power levels can be neglected if they are beyond P_M;

Energy-Efficient Transmission Algorithm

Find optimal total transmit power and its allocation that maximize Γ_{EE}

4. Calculate $\hat{\Gamma}_{\mathrm{EE}}(P_{J-1})$ and Λ_{J-1}, starting with $J = 1$ and stopping when Λ is negative;
5. If $\Lambda_0 \leq 0$, $P^{opt} = P_0$, go to Step 7;
6. When $\Lambda_{J-1} > 0$ and $\Lambda_J \leq 0$, the optimal total transmit power is in interval $(P_{J-1}, P_J]$. Establish Set $\mathcal{I}$. Find P^{opt} as the root of $\Theta(P)$ using the bisection method;
7. With P^{opt}, calculate the optimal transmit power allocation and the maximum energy efficiency.

Simulation Results

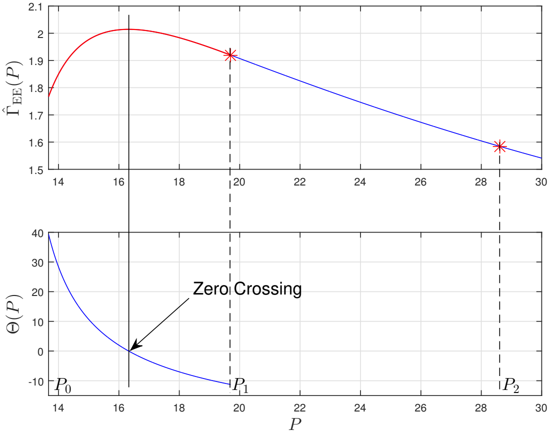


Figure: Search of the total transmit power that maximizes the energy efficiency. Fixed bandwidth assignment.

3. Joint Bandwidth Assignment and Power Allocation

The optimum sum rate $\hat{R}(W, P)$ is achieved as follows.

1. The $K - 1$ UEs with channel qualities $g_i, i = 2, 3, \ldots, K$, are transmitted to at their corresponding minimum required rates $R_i, i = 2, 3, \ldots, K$. All of the remaining resources of the spectrum and the transmit power is used for transmission to the one UE with the best channel quality g_1.

2. The maximum combined efficiency is

$$\left(\frac{1}{P} + \frac{\gamma}{W}\right) \hat{R}(W, P) =$$
$$\left(\frac{1}{P} + \frac{\gamma}{W}\right) \cdot \left(\sum_{i=2}^{K} R_i + \frac{w_1}{\ln 2} \left(\mathcal{W}_0\left(\frac{\psi g_1 - 1}{e}\right) + 1\right)\right)$$

where $\mathcal{W}_0(\cdot)$ is the principal branch of the Lambert W function.

Simulation Results

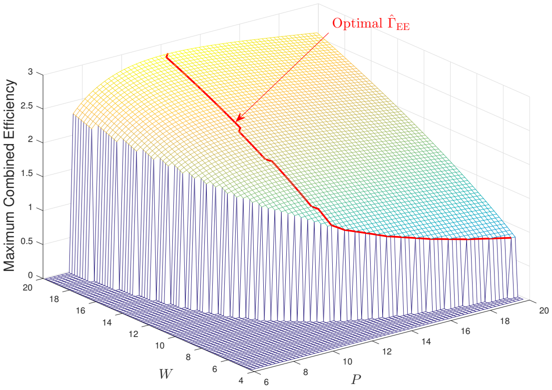


Figure: Maximum combined efficiency ($\approx \hat{\Gamma}_{\mathrm{EE}}$) as $\gamma = 0.01$. Solid red curve indicates the peak $\hat{\Gamma}_{\mathrm{EE}}$ with optimal total P.

Conclusion

- Over frequency-orthogonal broadcast channels, the problem of spectral- and energy-efficient transmission is formulated with maximum bandwidth and transmit power constraints and minimum rate requirements of the individual users.

- With a fixed transmit power allocation or a fixed bandwidth assignment, the problem is separated into two convex optimization problems. For each problem, the optimal bandwidth assignment or the optimal transmit power allocation is given by a water-filling solution.

- Effective procedures are provided to find total bandwidth W^{opt} that maximizes the spectral efficiency and total transmit power P^{opt} that maximizes the energy efficiency.